Write your student exam number in the boxes								

Manly Selective High School

2022 Higher School Certificate Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks: 100

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 38)

- Attempt Questions 11–36
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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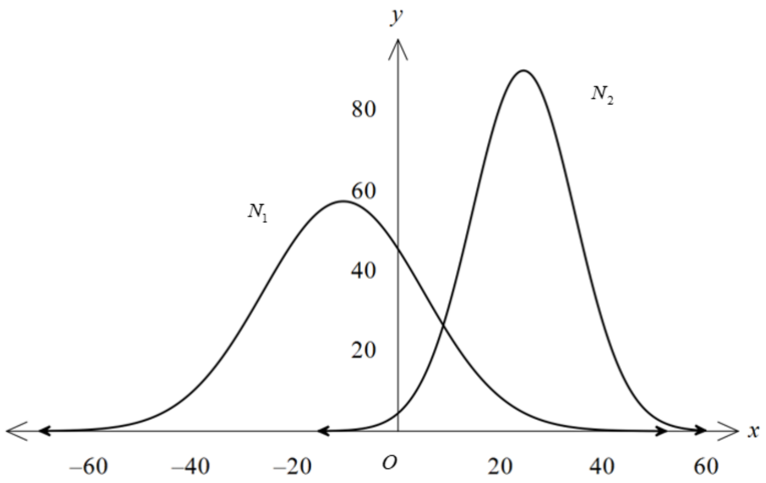
1. Consider the following graphs of two normal distributions, N_1 and N_2.



Assuming μ_1, σ_1 are the mean and standard deviation of N_1, while μ_2, σ_2, are the mean and standard deviation of N_2, which of the following statements is true?

A. $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$

B. $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$

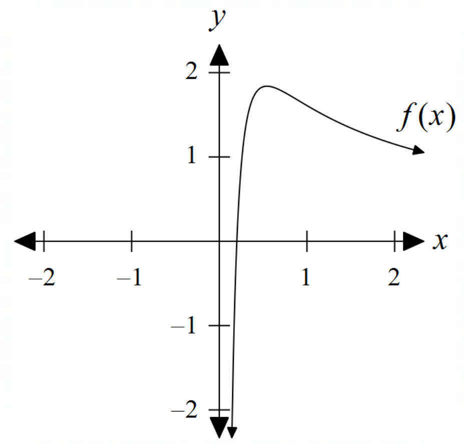
C. $\mu_1 < \mu_2$ and $\sigma_1 > \sigma_2$

D. $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$

2. The first term of an arithmetic sequence is 12. The fifteenth term of the same sequence is 67. What is the common difference of the sequence?

A. $\frac{11}{3}$ B. $\frac{55}{14}$ C. 3 D. $\frac{45}{14}$

3. The graph of $f(x)$ is shown.



Which of the following inequalities is true?

A. $f'(1) < f(1) < f''(1)$

B. $f(1) < f''(1) < f'(1)$

C. $f''(1) < f'(1) < f(1)$

D. $f'(1) < f''(1) < f(1)$

4. Which of the following expressions is equivalent to $\sec^2 x - \sin^2 x - \cos^2 x$?

A. $1 + \tan^2 x$

B. $1 - \tan^2 x$

C. $\tan^2 x - 1$

D. $\tan^2 x$

5. What is the 10th term of the sequence whose first 3 terms are 6144, 1536 and 384?

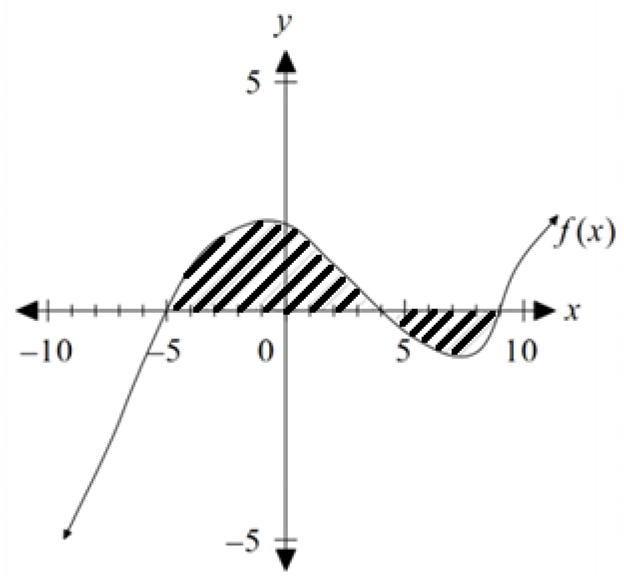
A. $\frac{3}{512}$

B. $\frac{3}{128}$

C. -35328

D. -39936

6. The curve shown in the graph has x-intercepts at -5, 4 and 9.



Which of the following would NOT give the shaded area?

A. $A = \left| \int_{-5}^{9} f(x)dx \right|$

B. $A = \int_{-5}^{4} f(x)dx + \left| \int_{4}^{9} f(x)dx \right|$

C. $A = \int_{-5}^{4} f(x)dx - \int_{4}^{9} f(x)dx$

D. $A = \int_{-5}^{4} f(x)dx + \int_{9}^{4} f(x)dx$

7. Let X and Y be two events such that $P(X) = 0.5$, $P(Y) = 0.6$ and $P(Y|X) = 0.7$
Which of the following statements is false?

A. $P(X|Y) < P(Y|X)$

B. X and Y are independent events.

C. $P(X \cap Y) = 0.35$

D. $P(X \cup Y) = 0.75$

8. Zane's grandparents put \$1200 in a savings account for him on the day of his birth and on every birthday until he turned 18. (The last deposit is made on his 18th birthday.) The account earns 2% interest compounding annually.

If Zane withdraws the money on his 18th birthday to pay for university, which expression gives the amount of money he will withdraw?

A. $\dfrac{1200(1.02^{18} - 1)}{0.02}$

B. $1200\ (1.02)^{18}$

C. $\dfrac{1200(1.02^{19} - 1)}{0.02}$

D. $1200(1.02)^{19}$

9. Let $f(x) = 2 - x$ and $g(x) = 6 + x$.

What is the domain of the function $h(x) = \ln\left\{\dfrac{f(-x)}{g(-x)}\right\}$?

A. $-2 < x < 6$

B. $-6 < x < 2$

C. $-\infty < x < 2 \cup 6 < x < 8$

D. $-\infty < x < 6 \cup 2 < x < \infty$

10. On a continuous function $f(x)$, h is a positive number which represents the horizontal difference between two points on the function.
Which of the following would NOT give the derivative of $f(x)$?

A. $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

B. $\lim_{h \to 0} \dfrac{f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)}{h}$

C. $\lim_{h \to 0} \dfrac{f(x) - f(x-h)}{h}$

D. $\lim_{h \to 0} \dfrac{f(x-h) - f(x)}{h}$

2022 Higher School Certificate Trial Examination
Manly Selective High School

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Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt questions 11 – 33
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 28 (64 marks)
Booklet 2 – Attempt Questions 29 – 33 (26 Marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 23. If you use this space, clearly indicate which question you are answering.

Question	Q 11 – 15	Q 16 – 18	Q 19 – 22	Q 23 – 25	Q 26 – 28	Total
Marks	/14	/12	/10	/14	/14	/64

Please turn over

Question 11 (4 marks)

Find:

a) $\int \sqrt[3]{(4x+7)}\, dx$ **2**

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b) $\int \frac{6(x-4)}{x^2-8x}\, dx$ **2**

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Question 12 (2 marks)

A series is given as $1 - 3k + 9k^2 + \ldots$

What is the value of k if the series has a limiting sum of $\frac{3}{4}$? **2**

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Question 13 (3 marks)

A sector has an arc length l of 2π cm and an area A of 24π cm^2.

The radius of the sector is r cm.

Find the central angle θ of the sector, in exact form. **3**

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Question 14 (2 marks)

Simplify fully: $(2\cos x + 3\sin x)^2 + (3\cos x - 2\sin x)^2$ **2**

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Question 15 (3 marks)

The share price ($\$P$) of a renewable energy company is increasing such that $\dfrac{dP}{dt} = \dfrac{3}{t+1}$,

where t is the time in months.

If the initial price of the share price is \$2.00, find the price after 6 months.

Write your answer to the nearest cent. **3**

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Question 16 (4 marks)

The table below show some values of two functions f and g, and of their derivatives f' and g'.

x	**1**	**2**	**3**	**4**
$f(x)$	4	6	-4	3
$g(x)$	2	0	-1	-5
$f'(x)$	4	8	3	7
$g'(x)$	-5	-4	-6	4

Calculate the following.

a) $\frac{d}{dx}(f(x)\ g(x))$ when $x = 4$ **2**

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b) $\int_1^3 (g'(x) + 5)\ dx$ **2**

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Question 17 (4 marks)

Consider the function $f(x) = \dfrac{4}{x+2} - 1$

a) Write down the equation of the vertical asymptote on the graph of $y = f(x)$. **1**

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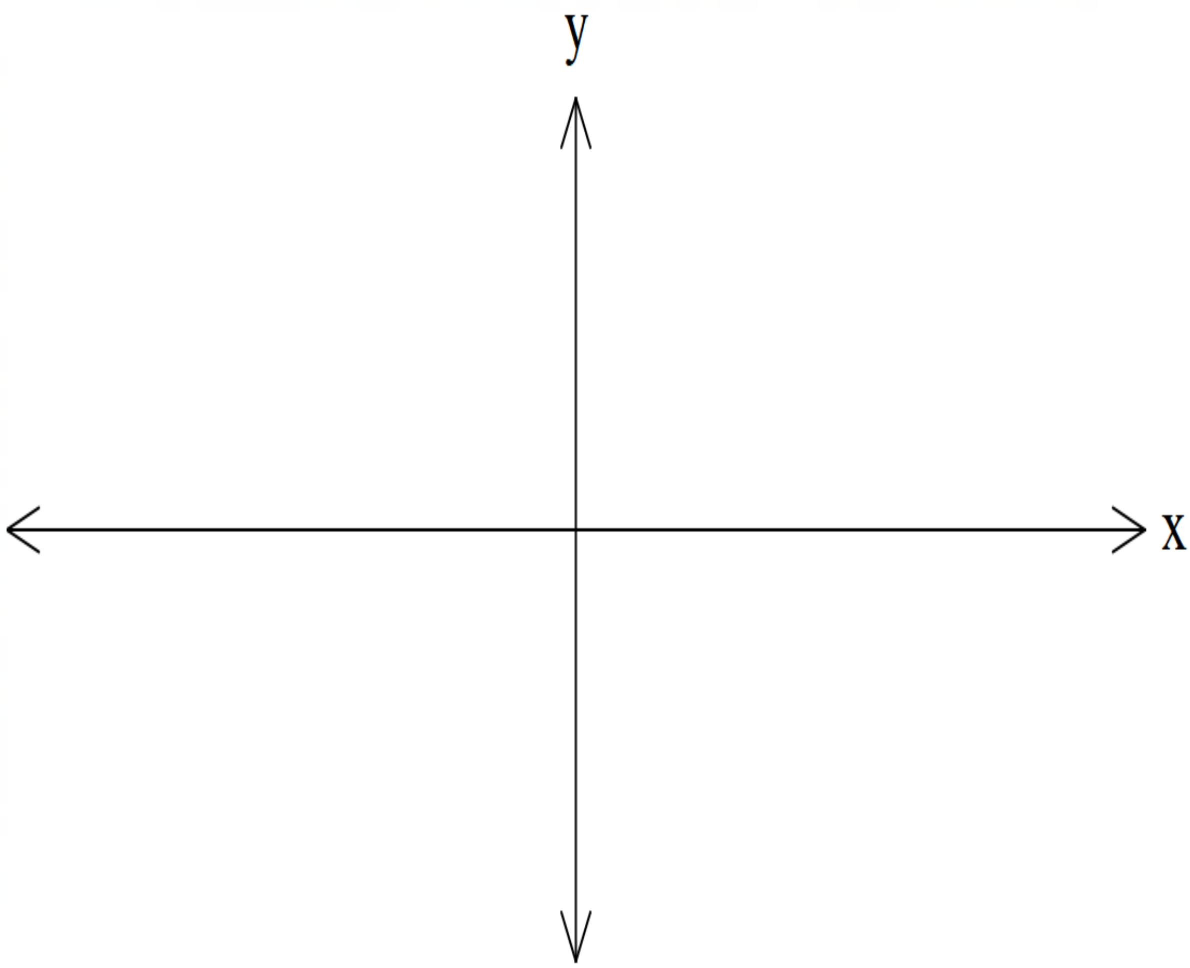
b) Sketch the graph of $y = f(x)$

Show clearly the asymptotes and the intercepts on the coordinate axes. **3**



Question 18 (4 marks)

Two six-sided dice are marked with numbers 1, 1, 2, 2, 3, 5 and 1, 3, 4, 5, 5, 5 respectively. The random variable X is defined to be the minimum (if the numbers are equal then that number is selected) of the two numbers obtained when the dice are thrown.

a) Fill in the table below giving the probability distribution of X. **2**

x	1	2	3	4	5
$P(X = x)$	$\frac{4}{9}$				

b) Find $P(X > 1)$ **1**

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c) Find Var(X) **1**

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Question 19 (2 marks)

Show that the derivative of $\frac{2x}{3x^2 + 1}$ is $\frac{2 - 6x^2}{(3x^2 + 1)^2}$ **2**

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Question 20 (2 marks)

The table below represents the future value of an annuity with a contribution of $1 at the end of each period.

Number of periods	Interest rate per period				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

Billy deposits $1000 into a savings account at the end of each month for 8 months.
The interest rate for these 8 months is 9% per annum, compounded monthly.

a) What will be the value of Billy's investment at the end of the 8-month period? **1**

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b) What would be the Present Value (PV) of this annuity? **1**

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Question 21 (3 marks)

A cylinder contains water at an initial depth of 10cm. Water is poured into the cylinder such that the height increases by 10cm in the 1st second, an additional 15 cm in 2nd second, an additional 20cm in the 3rd second and so on.

Assuming the cylinder has sufficient capacity, what would be the depth of water after 10 seconds. **3**

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Question 22 (3 marks)

a) Differentiate e^{x^2} **1**

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b) Hence or otherwise evaluate $\int_0^2 x e^{x^2}\, dx$ **2**

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Question 23 (7 marks)

Let $f(x) = x^3 + x^2 + 3x + 3$

a) Show that the graph of $y = f(x)$ has no stationary points. **2**

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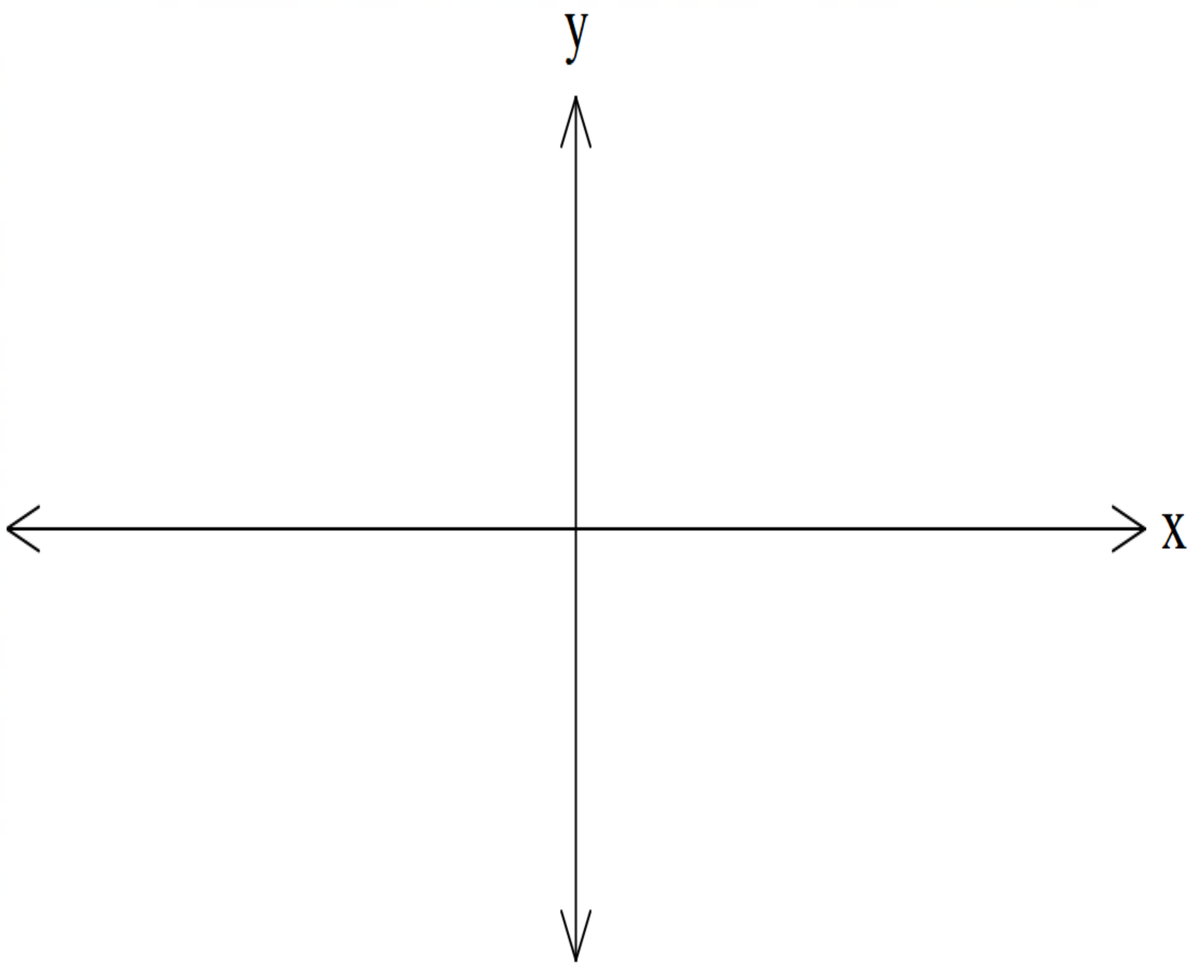
Question 23 (continued)

b) Find any points of inflexion. **2**

c) Sketch the graph $y = f(x)$, labelling any points of inflexion and all intercepts with the axes. **3**



Question 24 (4 marks)

A manufacturer stores drums of chemicals. During storage, evaporation takes place. A random sample of 10 drums was taken and the time in storage, x weeks, and the evaporation loss, y ml, are shown in the table below.

x = weeks	4	6	7	9	11	13	14	16	17	19
y = evaporation loss (ml)	35	49	52	60	68	78	81	89	87	95

a) Find the correlation coefficient r and comment on the strength of the relationship. **2**

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b) Find the equation of the least squares' regression line. **1**

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c) Using your model, predict the amount of evaporation that would take place after 35 weeks. Answer to the nearest mL. **1**

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Question 25 (3 marks)

In triangle ABC: AB = 12cm, BC = 20cm and AC = 28cm.

Find the exact area of the triangle. **3**

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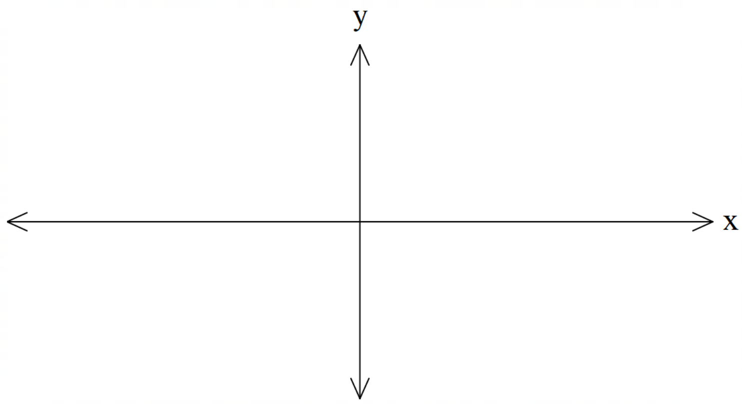
Question 26 (8 marks)

The exponential distribution of random variable X is defined by the probability density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$ where λ is a positive constant.

a) Sketch the graph of $y = f(x)$, showing any intercepts and asymptotes. **3**



b) Show that the cumulative distribution function is $F(x) = 1 - e^{-\lambda x}$. **2**

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Question 26 continues on the next page

c) Let the random variable X represent the wait time, in minutes, until a new customer enters a shop. The probability density function of X is exponential with $\lambda = \frac{1}{5}$.

i. Find the probability that the wait time until the next customer is more than 5 minutes, i.e. $P(X > 5)$. **1**

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ii. Show that if a customer has not entered the shop in the last 5 minutes, the probability that it will take at least another 5 minutes for a customer to enter remains the same as in part (c) (i).
That is, show $P(X > 10 \mid X > 5) = P(X > 5)$. **2**

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Question 27 (3 marks)

Solve $1 - \tan x = \sin x - \sin x \tan x$ for $[0, 2\pi]$ **3**

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Question 28 (3 marks)

The seats in a theatre are numbered in numerical order from the first row to the last row.

The first row has 14 seats. Each successive row has 4 more seats than the previous one.

If the theatre has a maximum capacity of 1320 seats, what is the greatest number of rows of seats in the theatre? **3**

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Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 29 – 33 (26 Marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 35. If you use this space, clearly indicate which question you are answering.

Question	Q 29	Q 30	Q 31	Q 32	Q 33	Total
Marks	/3	/4	/7	/7	/5	/26

Please turn over

Question 29 (3 marks)

A random variable Z has the standard normal distribution.

The table gives the probability of this random variable being greater than different values of z.

z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$P(Z > z)$	0.5	0.4013	0.3085	0.2266	0.1587	0.1057	0.668	0.0401	0.0228

In a hospital of 3125 patients, it is found that 708 patients have a body temperature higher than 36.8°C. The body temperatures are distributed normally with a mean μ and standard deviation 0.3°C.

Using the given table, find the value of μ, correct to two decimal places. **3**

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Question 30 (4 marks)

a) Find the centre and radius of the circle $x^2 - 2x + y^2 - 4y - 15 = 0$. **2**

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Question 30 continues on the next page

Question 30 (continued)

b) Determine the coordinates of the point on the circle for which the distance from the origin is a maximum. **2**

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Question 31 (7 marks)

John wants to accrue an investment of \$500 000 for his retirement in 30 years. He proposes to make a payment at the beginning of each year for the next 30 years.

The interest rate is set at 5% p.a. At the end of n years after starting his payments, the amount in the fund is A_n .

Let his first payment be \$$M$.

a) Show that $A_{30} = M \times 21 \times (1.05^{30} - 1)$ **3**

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b) Hence, calculate the annual payment M. **1**

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c) After 20 years, John is able to increase his payment to \$8000 per year. What will be the new value of his investment at the end of the 30-year period? **3**

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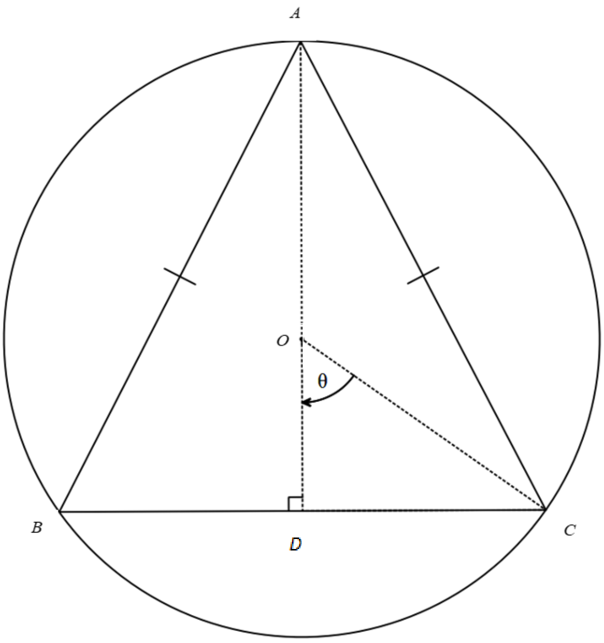
Question 32 (7 marks)

An isosceles triangle $\Delta\ ABC$ is inscribed inside a circle of fixed radius 2 and centre O. Let θ be defined as in the diagram below.



a) Show $DC = 2\sin\theta$. **1**

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b) Hence, or otherwise show that the area of the triangle $\Delta\ ABC$ is given by $A = 4\sin\theta(1+\cos\theta)$. **2**

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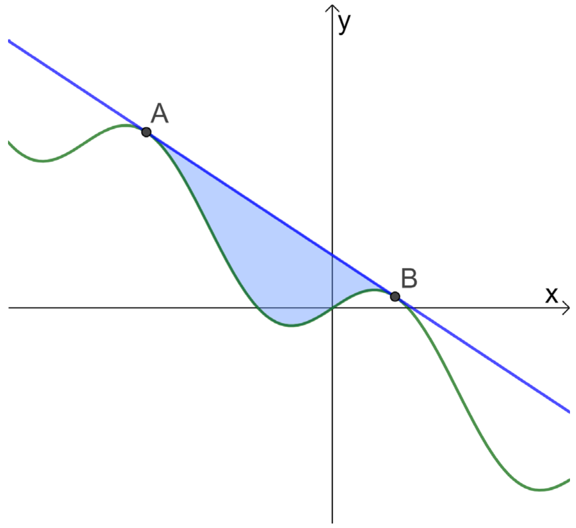
Question 32 continues on the next page

Question 32 (continued)

c) Find the value of θ that maximises the area of $\Delta\, ABC$. **4**

Question 33 (5 marks)

In the diagram, the line passing through the points A and B is tangent to the graph of $y = \sin 2x - x$ at A and B. The x-coordinate of point B is $\dfrac{\pi}{4}$.



Find the exact area of the shaded region. **5**

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End of paper

Question	Solution	Marks
1	$\mu_1 < \mu_2$ and $\sigma_1 > \sigma_2$	C
2	$67 = 12 + 14d$ $d = \frac{55}{14}$	B
3	$f(1) \approx 1.5$ $f\,'(1) < 0$ (decreasing) $f\,''(1) \approx 0$ (concavity changes) $\therefore f\,'(1) < f\,''(1) < f(1)$	D
4	$\sec^2 x - (\sin^2 x + \cos^2 x)$ $= \sec^2 x - 1$ $= \tan^2 x$	D
5	Geometric series with $r = 1536/6144 = 1/4$ $T_n = ar^{n-1} \rightarrow T_{10} = 6144\left(\frac{1}{4}\right)^9$ $= \frac{3}{128}$	B
6	$\int_{-5}^{10} f(x)\,dx$ gives the value of the definite integral from $x = -5$ to $x = 10$. The region from $x = 4$ to $x = 10$ is negative, so $\left\lvert \int_{-5}^{10} f(x)\,dx \right\rvert$ does not give the area.	A
7	False statement "X and Y are independent events" $P(Y\|X) = \frac{P(Y \cap X)}{P(X)}$ $\frac{P(Y \cap X)}{0.5} = 0.7$ $P(Y \cap X) = 0.35$ $P(Y) \times P(X) = 0.5 \times 0.6$ $= 0.3$ $P(Y) \times P(X) \neq 0.3$ $\therefore$ not independent	B

8	total $= 1200 + 1200(1.02) + 1200(1.02)^2 + \ldots + 1200(1.02)^{18}$ $= 1200(1 + 1.02 + 1.02^2 + \ldots + 1.02^{18})$ $= 1200\left(\dfrac{1.02^{19} - 1}{1.02 - 1}\right)$ $= 1200\dfrac{(1.02^{19} - 1)}{0.02}$	C
9	$f(-x) = 2 + x$ $g(-x) = 6 - x$ $h(x) = \ln\left(\dfrac{2 + x}{6 - x}\right)$ $h(x) = \ln(2 + x) - \ln(6 - x)$ $\therefore \quad 2 + x > 0 \Rightarrow x > -2$ and $6 - x > 0 \Rightarrow x < 6$ $\therefore \quad -2 < x < 6$	A
10	Option A is the definition of the derivative. Option B: the expression represents the gradient of the line joining $A\left(x - \frac{h}{2}, f\left(x - \frac{h}{2}\right)\right)$ and $B\left(x + \frac{h}{2}, f\left(x + \frac{h}{2}\right)\right)$. As h -> 0, this will give the gradient of the tangent at $(x,f(x))$. Option C: the expression represents the gradient of the line joining $(x - h, f(x - h))$ and $(x, f(x))$. As h->0, this will give the gradient of the tangent at $(x,f(x))$. Option D: the expression is equivalent to $-f'(x)$ (the opposite of Option C), which is not the same as $f'(x)$.	D
11a	$\int (4x + 7)^{\frac{1}{3}}\, dx$ $= \dfrac{(4x + 7)^{\frac{4}{3}}}{\frac{4}{3} \times 4}$ $= \dfrac{3}{16}(4x + 7)^{\frac{4}{3}} + c$	2 marks correct solution 1 mark partial progress to correct solution
11b	$3\int \dfrac{(2x - 8)}{x^2 - 8x}\, dx$ $= 3\ \ln\lvert x^2 - 8x\rvert + c$	2 marks correct solution 1 mark partial progress to correct solution

12	$a = 1$ $r = \dfrac{9k^2}{-3k}$ $= -3k$ $S_\infty = \dfrac{1}{1-(-3k)}$ $\dfrac{3}{4} = \dfrac{1}{1+3k}$ $3+9k = 4$ $k = \dfrac{1}{9}$	2 marks correct solution 1 mark partial progress to correct solution
13	$2\pi = r\theta$ $r = \dfrac{2\pi}{\theta}$ $24 = \dfrac{r^2\theta}{2}$ $48 = r^2\theta$ $48 = \dfrac{4\pi^2\theta}{\theta^2}$ $12\theta = \pi^2$ $\theta = \dfrac{\pi^2}{12}$	3 marks correct solution 2 marks partial correct with only one error 1 mark correct progress with formation of one correct equation
14	$4\cos^2 x + 9\sin^2 x + 12\cos x\sin x + 9\cos^2 x + 4\sin^2 x - 12\cos x\sin x$ $= 13\cos^2 x + 13\sin^2 x$ $= 13(\sin^2 x + \cos^2 x)$ $= 13$	2 marks correct solution 1 mark partial progress to correct solution
15	$P = \displaystyle\int \frac{3}{t+1}\,\mathrm{dt}$ $= 3\ \ln\lvert t+1\rvert\ + c$ for $t = 0, P = 2$ $\therefore\ 2 = 3\ln 1 + c$ $c = 2$ ie $P = 3\ \ln\lvert t+1\rvert + 2$ for $t = 6$ $P = 3\ln 7 + 2$ $= \$7.84$	3 marks correct solution 2 marks partial correct with only one error 1 mark correct progress with expression for P

<table>
<tr><td>16a</td><td>

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x) \text{ (product rule)}$$

when $x = 4$

$$\frac{d}{dx} = f(4)g'(4) + g(4)f'(4)$$

$= 3 \times 4 + -5 \times 7$

$= -23$

</td><td>2 marks – correct solution

1 mark – writes product rule and attempts to use values from table</td></tr>
<tr><td>16b</td><td>

$$\int_1^3 (g'(x) + 5\ dx) = [g(x) + 5x]$$

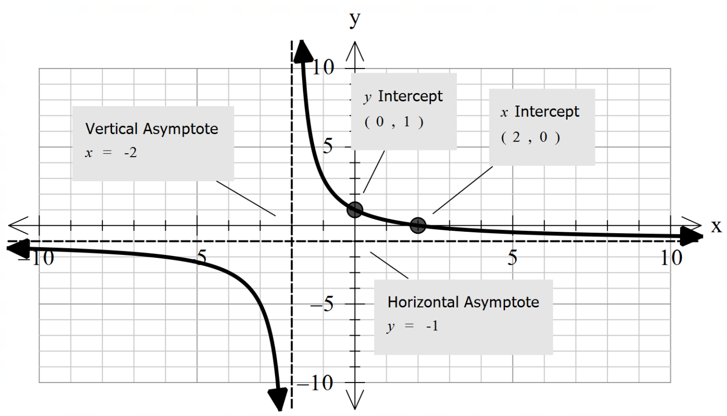
$= g(3) + 15 - (g(1) + 5)$

$= -1 + 15 - 2 - 5$

$= 7$

</td><td>2 marks – correct solution

1 mark – correct integration</td></tr>
<tr><td>17a</td><td>$x = -2$</td><td></td></tr>
<tr><td>17b</td><td>



</td><td>3 marks – all intercepts and asymptotes correct

2 marks – 1 feature missing

1 mark – 2 or 3 features missing</td></tr>
<tr><td>18a</td><td>

<table>
<tr><th></th><th>1</th><th>1</th><th>2</th><th>2</th><th>3</th><th>5</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>3</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>3</td></tr>
<tr><td>4</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>5</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>5</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>5</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>5</td></tr>
</table>

Tabulate the minimum values of the 2 dice as above. And hence complete the given table as required:

<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td></td></tr>
<tr><td>P(X=x)</td><td>16/36
=4/9</td><td>10/36
=5/18</td><td>6/36=
1/6</td><td>1/36</td><td>3/36=
1/12</td><td></td></tr>
</table>

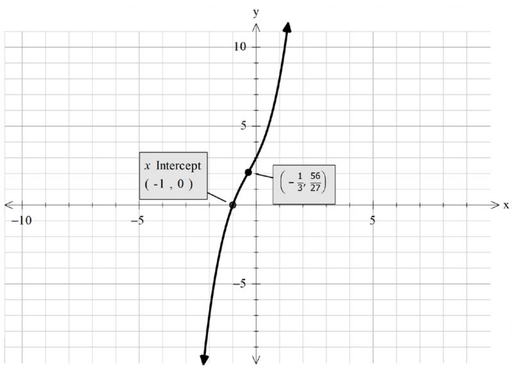
</td><td>2 marks – correct values in table with working shown

1 mark – progress towards correct solution</td></tr>
</table>

18b	1-4/9 =5/9	1 mark – correct answer
18c	From calculator $\sigma(x) = 1.213$ $\text{Var}(x) = \sigma^2 = 1.471$	1 mark – correct answer
19	$y = \dfrac{2x}{3x^2 + 1} \rightarrow u = 2x,\ v = 3x^2 + 1$ $\therefore u' = 2,\ v' = 6x$ $y' = uv' + vu' \rightarrow y' = \dfrac{2(3x^2 + 1) + 2x.(3x)}{(3x^2 + 1)^2}$ $y' = \dfrac{6x^2 + 2 - 12x^2}{(3x^2 + 1)^2}$ $y' = \dfrac{2 - 6x^2}{(3x^2 + 1)^2}$	2 Marks: Correct answer 1 Mark: Correct expressions for du/dx and dv/dx
20a	1000 x 8.2132= 8213.20	1 Mark: Correct answer
20b	$P(1 + r)^n = 8213.20 \rightarrow P = \dfrac{8213.2}{(1.0075)^8}$ $P = 7736.63$	1 Mark: Correct answer
21	Initial depth = 10 *cm* Height increase = 10, 15, 20, …… $a = 10 \quad d = 5 \quad n = 10$ $S_n = \dfrac{n}{2}\{2a + (n - 1)d\} \rightarrow S_{10} = \dfrac{10}{2}\{20 + (10 - 1).\ 5$ $= 325\ cm$ Final depth $= 325 + 10 = 335\ cm$	3 Marks: Correct answer 2 Marks: Calculates correct increase in depth of water 1 Mark: Finds T_{10} instead of S_{10}, OR finds S_{10} for the series 20, 35 50, or similar.
22a	$y = e^{x^2} \rightarrow \dfrac{dy}{dx} = e^{x^2}.\ 2x = 2xe^{x^2}$	1 Mark: Correct answer
22b	$\int_0^2 xe^{x^2}\ dx = \left(\dfrac{1}{2}\right)\int_0^2 2xe^{x^2}\ dx = \left(\dfrac{1}{2}\right)[e^{x^2}]$ from $x = 0$ to $x = 2$. $\int_0^2 xe^{x^2}\ dx = \dfrac{1}{2}[e^4 - e^0] = \dfrac{1}{2}[e^4 - 1]$	2 Marks: Correct answer 1 Mark: Correct primitive function
23a		

<table>
<tr><td></td><td>$f(x) = x^3 + x^2 + 3x + 3$
$f'(x) = 0$ for SPs
$3x^2 + 2x + 3 = 0$
has no solutions
as $\Delta = 2^2 - 4(3)(3)$
$= -32$
$f(x)$ has no stationary points</td><td>2 marks correct solution

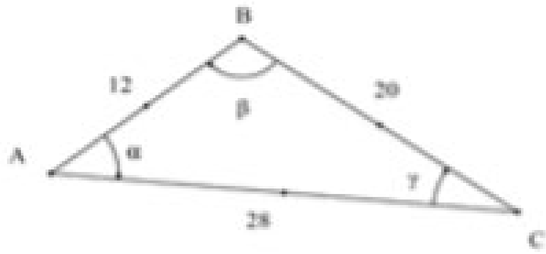
1 mark correct f'(x)</td></tr>
<tr><td>23b</td><td>$f''(x) = 6x + 2$
$f''(x) = 0$
$6x + 2 = 0$
$x = -\frac{1}{3}$
$f\left(-\frac{1}{3}\right) = \frac{56}{27}$
<table><tr><td>x</td><td>-1</td><td>$-\frac{1}{3}$</td><td>0</td></tr><tr><td>$f''(x)$</td><td>-4</td><td>0</td><td>2</td></tr></table>
Since $f''(x)$ changes sign, point of inflexion at $\left(-\frac{1}{3}, \frac{56}{27}\right)$</td><td>2 marks correct solution

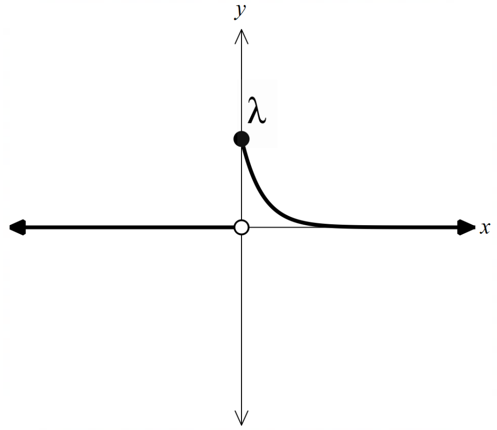
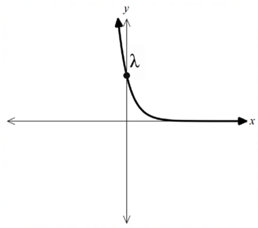
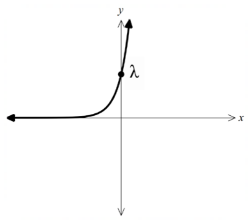
1 mark correct
x value no change in concavity shown</td></tr>
<tr><td>23c</td><td>
</td><td>3 marks correct solution

2 marks correct graph showing correct concavity and point of inflexion

1 mark correct intercepts and correct shape</td></tr>
<tr><td>24a</td><td>r= 0.9901098(by calculator) Strong linear relationship</td><td>2 marks correct solution
1 mark correct r
Or correct description</td></tr>
<tr><td>24b</td><td>by calculator A = 24.12 B = 3.90
$y = 3.90x + 24.12$</td><td>1 mark correct equation</td></tr>
<tr><td>24c</td><td>$y = 3.9(35) + 24.12$
$= 160.8$

(Using stored exact values for A & B)</td><td>1 mark correct answer
Or cfpe from b)</td></tr>
</table>

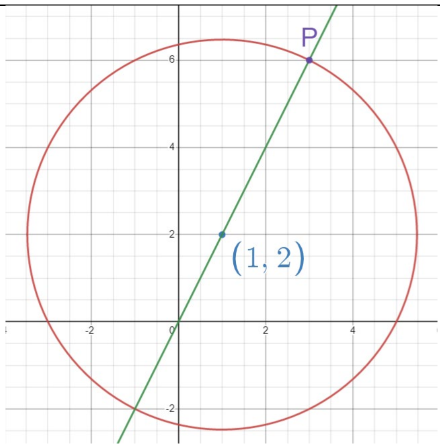
25	 $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $\cos\beta = \frac{12^2 + 20^2 - 28}{2 \times 12 \times 20}$ $= -\frac{1}{2}$ $\beta = 120°$ $A = \left(\frac{1}{2}\right) \times (12)(20)\sin(120)$ $= 120\frac{\sqrt{3}}{2}$ $= 60\sqrt{3}\ \text{cm}^2$ or	3 marks correct answer 2 marks Correct exact sine ratio or correct area approximation. Sin-1 notation etc not allowed 1 mark Correct exact cosine ratio for non-right triangle

	$\cos\alpha = \dfrac{12^2 + 28^2 - 20^2}{2 \times 12 \times 28}$ $= \dfrac{11}{14}$ $\sin\alpha = \dfrac{5\sqrt{3}}{14}$ by Pythagoras $Area = \dfrac{1}{2} \times (12)(28)\sin\alpha$ $= 6 \times 28 \times \dfrac{5\sqrt{3}}{14}$ $= 60\sqrt{3}$	
26a		3 marks 1 mark for correct exponential curve 1 mark for correct intercept 1 mark for $y = 0$ <u>Note:</u> 2 marks for:  1 mark for 
26b	For a PDF $f(x)$ with domain $a \leq x \leq b$, the CDF is given by: $P(X \leq x) = F(x) = \displaystyle\int_a^x f(x)dx$	2 marks 1 mark for a correct integral expression for F(x) or 1 mark for correct primitive

	$F(x) = \int_0^x \lambda e^{-\lambda x}\,dx$ $= [-e^{-\lambda x}]_0^x$ $= -e^{-\lambda x} + 1$ $= 1 - e^{-\lambda x}$ Or more formally: $P(X \geq x) = F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$	
26ci	$F(x) = 1 - e^{-\frac{x}{5}}$ $P(X > 5)$ $= 1 - P(X < 5)$ $= 1 - \left(1 - e^{\left(-\frac{5}{5}\right)}\right)$ $= 1 - (1 - e^{-1})$ $= \frac{1}{e}$	1 mark
26cii	$P(X > 10) = 1 - \left(1 - e^{-\frac{10}{5}}\right)$ $= 1 - (1 - e^{-2}) = e^{-2} = \frac{1}{e^2}$ $P(X > 10 \mid X > 5)$ $= \frac{P(X > 10 \text{ and } X > 5)}{P(X > 5)}$ $= \frac{P(X > 10)}{P(X > 5)}$ $= \frac{1}{e^2} \div \frac{1}{e}$ $= \frac{1}{e}$	2 marks 1 mark for finding $P(X > 10)$ or 1 mark for dividing by P(X > 5) (1/e)

27	$$1 - \tan x = \sin x - \sin x \tan x$$ $$\sin x - \sin x \tan x - 1 + \tan x = 0$$ $$\sin x(1 - \tan x) - (1 - \tan x) = 0$$ $$(\sin x - 1)(1 - \tan x) = 0$$ $$\sin x = 1 \text{ or } \tan x = 1$$ For $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$ But this does not satisfy the original equation, because $\tan\frac{\pi}{2}$ is undefined. $\therefore \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$ **Marker's comment:** Most students found this difficult. 1) A note about cancelling Many students did this: $1 - \tan x = \sin x - \sin x \tan x$ $1 - \tan x = \sin x(1 - \tan x)$ $1 = \sin x$ The problem with dividing both sides by $(1 - \tan x)$ is we are assuming $1 - \tan x \neq 0$. The correct approach is to bring all terms to the LHS, then factorise and use the Null Factor Law, as above. Follow-up question: Solve $x^2 = 2x$. (Would you cancel x initially?) 2) Checking solutions by substitution $x = \pi/2$ does not satisfy the original equation, because $\tan\pi/2$ is undefined. Beware of equations which contain terms like $\tan x$, $\log x$, $1/x$, $\sqrt{x}$ etc., which are undefined for some values of x. In general, you can always check your final answers by substituting back into the original equation. Follow-up questions: Solve $\sqrt{x} = x - 2$. Solve $\log_5 (x^2 - 4x) = \log_5(x - 4) + 1$.	3 marks 1 mark for factorising (taking out $\sin x$ as a common factor, or equivalent) 2 marks for finding $\frac{\pi}{2}$ and then ruling it out 2 marks for three solutions $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}$

28	$S = 14 + 18 + \ldots$ $a = 14,\ d = 4$ $S_n = \frac{n}{2}[2a + (n-1)d)]$ $S_n = \frac{n}{2}[2(14) + 4(n-1)]$ $S_n = \frac{n}{2}(24 + 4n) = 12n + 2n^2 \leq 1320$ $2n^2 + 12n - 1320 \leq 0$ $n^2 + 6n - 660 \leq 0$ $n = \frac{-6 \pm \sqrt{6^2 - 4(1)(-660)}}{2(1)}$ $n = -28.8 \text{ or } 22.8$ $\therefore \quad S_n \leq 1320 \text{ if } n \leq 22.8$ $\therefore \quad n = 22$ is the max number of rows <u>Note</u>: S_{23} is (a bit) more than 1320, S_{22} is (a bit) less than 1320. So the maximum number of (complete) rows is 22.	3 marks 1st mark for correct expression for S*n* 2nd mark for correct expression for *n* 3rd mark for correct answer (by rounding down) <u>Note:</u> Some students used T_n formula instead of S_n. This yields a linear equation instead of a quadratic which makes the question easier, so only 1 mark was awarded.
29	$P\left(temp > 36.8^0\right)$ $= \frac{708}{3125}$ $= 0.2266$ From the table, $P(Z > 0.75) = 0.2266$ $\frac{x-\mu}{\sigma} = Z$ $\frac{36.8-\mu}{0.3} = 0.75$ $\mu = 36.8 - 0.3(0.75)$ $= 36.58$	3 marks correct solution 2 marks relates Z score of the table 1 mark correct P(T>36.8)
30a	$x^2 - 2x + y^2 - 4y = 15$ $(x-1)^2 + (y-2)^2 = 15 + 1 + 4$ $(x-1)^2 + (y-2)^2 = 20$ centre: $(1,2)$, radius $= 2\sqrt{5}$	2 marks
30b	Let *P* be the point on the circle furthest away from the origin:	2 marks



(Note: O(0,0), C(1,2) and P must be collinear – if not, then the direct distance OP would be shorter than OC + CP.]

The equation of the diameter through the centre and P is $y = 2x$

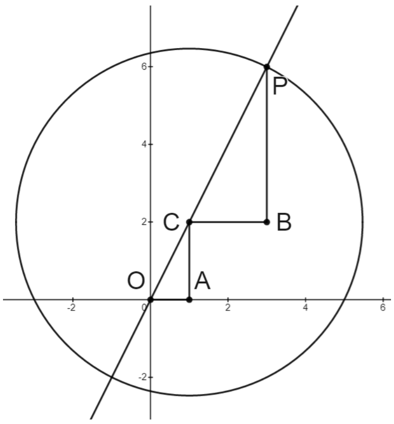
Find points of intersection between $y = 2x$ and the circle by solving simultaneously:

$$x^2 - 2x + (2x)^2 - 4(2x) = 15$$
$$x^2 - 2x + 4x^2 - 8x = 15$$
$$5x^2 - 10x = 15$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = 3 \text{ or } -1$$

when $x = 3,\ y = 6$

$\therefore \quad P = (3,6)$

<u>Alternative method:</u>



1 mark for attempting to find the point of intersection between $y = 2x$ and the circle

or

1 mark for using calculus to find the stationary point on the circle (although this is irrelevant)

or

1 mark for forming an equation for the distance OP (e.g. $x^2 + y^2 = 45$)

	$OC = \sqrt{1^2+2^2} = \sqrt{5}$ (Pythag) $PC = 2\sqrt{5}$ (radius) $\tan \angle COA = \dfrac{AC}{OA} = \dfrac{2}{1}$ $\angle COA = \tan^{-1}2\ (\approx 63.43\ldots)$ $\therefore \quad \angle PCB = \tan^{-1}2$ (corresponding angles on parallel lines) $\sin \angle PCB = \dfrac{PB}{2\sqrt{5}}$ $PB = 2\sqrt{5}\sin\left(\tan^{-1}2\right) = 4$ similarly, $BC = 2\sqrt{5}\cos\left(\tan^{-1}2\right) = 2$ $\therefore \quad P = (1+2,\ 2+4) = (3,6)$	
31a	$r = 5\%\ pa \rightarrow R = 1 + r = 1.05$ $A_1 = M(1.05)$ $A_2 = [M(1.05) + M](1.05) = M(1.05)^2 + M(1.05$ $A_3 = [M(1.05)^2 + M(1.05) + M](1.05)$ $= M(1.05)^3 + M(1.05)^2 + M(1.05)$ . . . $A_{30} = M[1.05 + 1.05^2 + 1.05^3 + \ldots + 1.05^{30}]$ $= M\left\{(1.05)\dfrac{1.05^{30} - 1}{1.05 - 1}\right\}$ $= M\left(\dfrac{1.05}{0.05}\right)\{1.05^{30} - 1\}$ $A_{30} = M(21)\{1.05^{30} - 1\}$	3 Marks: Correct answer 2 Marks: Obtains correct expressions for A_1, A_2 and A_3 and attempts to form an expression for A_{30}. OR Starts at a correct expression for A_{30} and correctly simplifies. 1 Mark: Obtains correct expressions for A_1 and A_2 OR Forms a correct expression for A_{30}
31b	$M = \dfrac{500000}{21(1.05^{30} - 1)}$ $= 7167.35$	1 Mark: Correct answer

31c	$A_n = M(21)(1.05^n - 1)$ $A_{20} = 7167.35(21)(1.05^{20} - 1) = \$248845.03$ $A_n = PR^n \rightarrow A_{10} = \$248845.03(1.05)^{10}$ $A_{10} = \$405342.33$ For $M = \$8000$, $A_{10} = 8000(21)(1.05^{10} - 1)$ $A_{10} = \$105654.30$ $\therefore$ Total $= \$405342.33 + \$105654.30$ $= \$510996.63$	3 Marks: Correct answer 2 Marks: Obtains correct values for two of A_{10}, A_{20} or A_{30} 1 Mark: Obtains A_{10} or A_{20}
32a	In ΔODC $\sin\theta = \frac{DC}{OC}$ $\sin\theta = \frac{DC}{2}$ (OC is radius) $\therefore DC = 2\sin\theta$	1 mark – clearly shown
32b	$A = \frac{1}{2}$ base $\times$ height where base = $BC = 4\sin\theta$ and height = $OD + OA = 2\cos\theta + 2$ $A = \frac{1}{2} \times 4\sin\theta \times (2 + 2\cos\theta)$ $A = 4\sin\theta(1 + 1\cos\theta)$	2 marks – correct solution fully detailed. 1 mark –
32c	$\frac{dA}{d\theta} = 4\cos\theta(1 + \cos\theta) - 4\sin\theta\sin\theta$ $= 4\cos^2\theta - 4\sin^2\theta + 4\cos\theta$ $= 4(\cos^2\theta - \sin^2\theta + \cos\theta)$ $= 4(\cos^2\theta - (1 - \cos^2\theta) + \cos\theta)$ $= 4(2\cos^2\theta + \cos\theta - 1)$ $= 4(2\cos\theta - 1)(\cos\theta + 1)$ $\frac{dA}{d\theta} = 0$ when $\cos\theta = -1, \frac{1}{2}$ $\theta = 180°, 60°$ as θ is acute $\theta = 60°$ Check its a max θ: 50°, 60°, 70° $\frac{dA}{d\theta}$: 0.719, 0, -1.69 $\therefore$ max Area when $\theta = 60°$	4 marks – correct solution fully detailed 3 marks – correct value but doesn't check it's a max 2 marks – applies trig identities and factorises correctly 1mark – differentiates correctly

33	Eqn of line $f'(x) = 2\cos 2x - 1$ $f'\left(\frac{\pi}{4}\right) = 2\cos\left(\frac{\pi}{2}\right) - 1$ $= -1$ $f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}\right) - \frac{\pi}{4}$ $= 1 - \frac{\pi}{4}$ $y - \left(1 - \frac{\pi}{4}\right) = -\left(x - \frac{\pi}{4}\right)$ $y = 1 - x$ Pts of intsn $1 - x = \sin 2x - x$ $\sin 2x = 1$ $2x = \frac{\pi}{2}$ or $-\frac{3\pi}{2}$ $x = \frac{\pi}{4}$ or $-\frac{3\pi}{4}$ $A = \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} (1 - x) - (\sin 2x - x)\ dx$ $= \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} 1 - \sin 2x$ $= \left[x + \frac{1}{2}\cos 2x\right]_{-\frac{3\pi}{4},\frac{\pi}{4}}$ $= \left(\frac{\pi}{4} - \frac{1}{2}\cos\frac{\pi}{2}\right) - \left(-\frac{3\pi}{4} - \frac{1}{2}\cos\frac{3\pi}{2}\right)$ $= \frac{\pi}{4} + \frac{3\pi}{4}$ $= \pi$	5 marks correct solution 4 marks correct progress to solution up to correct antiderivative for area with endpoints 3 marks correct progress to solution up to correct x coordinate of point A 2 marks correct progress to solution up to correct equation of the tangent 1 mark correct progress to solution up to correct gradient of the tangent